

ASYMMETRIC IMPACT OF INVESTMENT AND LABOUR FREEDOMS ON THE INFORMAL ECONOMY: EVIDENCE FROM NIGERIA

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Abstract

This study examines the asymmetric relationship between economic freedoms and informal sector dynamics in Nigeria using a Nonlinear Autoregressive Distributed Lag (N-ARDL) approach. The analysis employs dynamic general equilibrium (DGE) model-based estimates of informal sector size to evaluate long-run and short-run asymmetries while accounting for macroeconomic and institutional factors. Wald test results confirm significant asymmetries in both temporal dimensions. The empirical results demonstrate that investment freedom changes generate pronounced asymmetric effects: a 1% increase reduces informal sector size by 0.15%, while a 1% decrease expands it by only 0.08%, indicating a "ratchet effect" in formalization. Labour freedom demonstrates similar but more modest asymmetry, with a 1% increase and decrease yielding -0.10% and +0.06% effects, respectively. This pattern aligns with theoretical expectations that positive regulatory changes elicit stronger responses than negative ones, with investment freedom showing greater asymmetry due to higher responsiveness of capital flow to favourable conditions compared to labour market adjustments. The analysis of control variables indicates that GDP per capita growth (-0.20%) and institutional quality (-0.30%) constrain informality, while unemployment (0.35%) and urbanization (0.50%) expand it. The error correction term (-0.52) indicates rapid equilibrium adjustment. The Practical policy implications of the findings are discussed.

Keywords: Informal Economy; Investment Freedom; Labour Freedom; Asymmetry.

1. INTRODUCTION

The informal sector, also known as the informal economy encompasses a wide range of economic activities that are not regulated or protected by governmental authorities. This

includes unregistered businesses, informal employment, and various forms of self-employment that do not adhere to formal labour laws or tax regulations (Barra & Papaccio, 2024). The informal sector plays a crucial role in many economies, particularly in developing countries, where it often serves as a primary source of income for millions of people. The informal economy comprises more than 60% of total employment in Sub-Saharan Africa, with estimates suggesting that 80% to 95% of the working population in many countries rely on informal labour (Friedrich-Ebert-Stiftung [FES], 2022). In Nigeria, the focus of this study and a representative developing economy, it accounted for about 65% of the GDP in 2017, highlighting its importance as a source of employment and income (Bank of Industry [BOI], 2022). Globally, the informal economy can represent up to one-third of economic activity in low- and middle-income countries (Ghorpade et al., 2024). However, the forces governing the growth of this sector, particularly in relation to economic freedoms, remain inadequately understood.

Investment freedom and labour freedom are two fundamental components of economic freedom that significantly influence the trajectory of informal sector growth (Saunoris, 2024). Investment freedom encompasses the capacity of individuals and firms to allocate and reallocate capital and resources across various economic activities, both domestically and internationally, without undue governmental or institutional constraints. This freedom is crucial for efficient resource allocation and economic dynamism. Labour freedom, conversely, pertains to the regulatory environment of a country's labour market. It encompasses a wide range of factors, including but not limited to minimum wage legislation, employment protection laws, hiring and firing regulations, working hour restrictions, and severance pay requirements. The degree of labour freedom in an economy can profoundly impact labour market flexibility and, consequently, the size and dynamics of the informal sector.

The theoretical relationship between these economic freedoms and informal sector growth is complex and multidimensional. Classical economic theory posits that enhanced economic freedoms should catalyze a contraction of the informal sector by fostering increased opportunities and reduced barriers to entry in the formal economy. This perspective suggests that as constraints on investment and labour markets are relaxed, the relative attractiveness of formal sector participation increases, thereby incentivizing informal enterprises and workers to transition into the formal economy (European Commission et al., 2024).

Despite a growing body of literature on the informal economy and economic freedoms, there remains a significant research gap in understanding their interplay, particularly in developing economies. Most existing studies have assumed a symmetric relationship between economic freedoms and informal sector growth, potentially overlooking important distinctions. Furthermore, the specific case of Nigeria has not been thoroughly examined through this lens. Against this backdrop, this research aims to address these gaps by answering the following questions: Does the informal sector in Nigeria respond more strongly to decreases in investment freedom compared to increases of equal

magnitude? Is the growth of Nigeria's informal sector more sensitive to reductions in labour freedom than to equivalent increases? Is the asymmetry between positive and negative changes more pronounced for labour freedom compared to investment freedom in terms of their effects on informal sector growth in Nigeria?

By addressing these questions, this study contributes to the existing body of knowledge in several ways. Firstly, it provides empirical evidence on the potentially asymmetric impacts of economic freedoms on informal sector growth, offering a more comprehensive understanding of this relationship. Secondly, it focuses on Nigeria as a case study, providing insights that may be applicable to other developing economies with similar characteristics. Lastly, the study's findings can inform policy decisions aimed at managing informal sector growth and promoting overall economic development.

The relevance of this research extends beyond academic circles. Policymakers in developing economies often grapple with the challenge of balancing formal sector growth with the realities of large informal sectors. Understanding the potentially asymmetric impacts of economic freedoms can lead to more targeted and effective policies that consider both the positive and negative consequences of regulatory changes.

2. LITERATURE REVIEW

The relationship between economic freedoms and the informal sector has garnered increasing attention in economic literature, particularly in the context of developing economies. However, studies specifically examining the influence of investment freedom and labour freedom on the informal sector in these economies remain limited, and research on their potential asymmetric impacts is notably absent. This review synthesizes the existing body of knowledge on the broader impacts of economic freedoms on informal sector growth, while identifying gaps in the understanding of asymmetric effects and the specific roles of investment and labour freedoms.

The general relationship between economic freedom and the size of the informal or shadow economy has been explored by several researchers. Koyuncu and Ünal (2019) examined this relationship across 153 countries from 1999 to 2013, finding that overall economic freedom and business freedom had the strongest negative association with the size of the shadow economy. Interestingly, they noted that investment freedom had the least impact among the freedom indicators studied, though the reasons for this were not fully explored. Building on this, D'Agostino et al. (2023) analyzed data from 152 countries between 1995 and 2017, revealing a more complex relationship. They found that the effect of economic freedom on the shadow economy is U-shaped, primarily driven by business regulation and the freedom in the legal system and property rights. Importantly, they observed that the negative effect of economic freedom on the shadow economy was more pronounced in countries with low levels of democracy and high corruption, suggesting the importance of institutional context in shaping these relationships.

Berdiev et al. (2018) provided further support for the negative relationship between economic freedom and the shadow economy. Their study of over 100 countries from 2000 to 2015 found that all aspects of economic freedom significantly mitigate shadow activities, with freedom from regulation exhibiting the largest impact. However, like many studies in this area, they did not specifically focus on developing economies or examine the potential for asymmetric effects of different components of economic freedom.

While these studies establish a general relationship between economic freedom and the informal sector, they do not fully address the potential for asymmetric effects, particularly in the context of developing economies. Baez (2014) provided early evidence of non-linear effects, finding that informal labour markets positively affect FDI flows up to a certain level of informality, above which the effect becomes negative. This suggests the possibility of asymmetric impacts depending on the initial conditions of the economy, but the study did not explicitly examine the roles of investment and labour freedoms.

Freytag et al. (2022) directly addressed the issue of asymmetry in their study of OECD and other European countries. They found that the effects of changes in economic freedom on the shadow economy are indeed asymmetric, with stronger effects observed when economic freedom increases compared to when it decreases. This finding underscores the importance of considering directional changes in economic freedom when analyzing its impact on the informal sector. However, their focus on developed economies limits the applicability of their findings to developing country contexts.

The specific role of investment freedom in shaping the informal sector has received less attention in the literature, particularly in developing economies. Bhavan (2023) examined the relationship between foreign direct investment (FDI) and the informal sector in Sri Lanka, finding a negative and significant impact of FDI on the informal sector. This suggests that increased investment freedom, which often accompanies increased FDI, may discourage informal sector growth. However, the study did not explicitly consider the asymmetric nature of this relationship or extend its analysis to other developing economies. The impact of labour freedom on the informal sector has been more extensively studied, though again with limited focus on developing economies and potential asymmetries. Das and Kapoor (2024) found a positive and significant relationship between the intensity of labour freedom and the land-based status of peasant households in rural India. Their study highlights the importance of labour market reforms, particularly those focused on wage contracts and minimum wage laws, in enhancing labour freedom and potentially reducing informality. However, Adair et al. (2024) provided a contrasting perspective, suggesting that informal employment in middle-income MENA countries is not driven by choice on the labour supply side but by structural constraints on the demand side. This implies that labour freedom alone may not be sufficient to reduce informality if other structural barriers persist, highlighting the complex nature of these relationships in developing economies.

Recent literature has increasingly recognized the importance of contextual factors in shaping the relationship between economic freedoms and the informal sector. Saunoris

(2024) examined the interplay between formal institutions, culture, and the size of the shadow economy, finding that economically free countries with individualistic cultures tend to have smaller shadow economies. Thu et al. (2024) further expanded this contextual understanding by examining the relationships between social globalization, political globalization, economic freedom, and the shadow economy in ASEAN countries. Their findings demonstrate that both de facto and de jure globalization can mitigate the proliferation of the shadow economy, suggesting that the impacts of economic freedoms should be considered within the broader context of globalization.

Despite these valuable insights, a significant gap remains in understanding the potentially asymmetric effects of investment and labour freedoms on informal sector growth, particularly in developing economies. Most existing studies have assumed symmetric effects or have not explicitly tested for asymmetry. Furthermore, the specific case of Nigeria, as a representative developing economy, has not been thoroughly examined through this lens. The limited focus on investment freedom and labour freedom in the context of informal sector dynamics in developing economies is particularly notable, given their potential importance in shaping economic outcomes in these contexts. Against this backdrop, this study aims to address these crucial gaps by explicitly modeling and testing for asymmetric effects of investment and labour freedoms on informal sector growth in Nigeria. The novel approach of examining asymmetric impacts in this context promises to shed new light on the dynamics between economic freedoms and informal sector growth, potentially challenging existing assumptions and informing more effective policy interventions.

3. THEORETICAL UNDERPINNINGS, DATA AND METHOD

3.1 Theoretical Underpinnings

The relationship between economic freedoms and informal sector growth is complex, particularly in developing economies like Nigeria. This study focuses on the asymmetric impacts of investment and labour freedoms on informal sector growth, drawing primarily from institutional economics and labour market theory.

3.1.1 *Institutional Economics and Asymmetric Effects*

North (1990) posits that institutions, both formal and informal, are the "rules of the game" that shape economic behavior and outcomes. In the context of this study, investment and labour freedoms represent institutions that significantly influence economic actors' decisions. These freedoms, or the lack thereof, create incentives and disincentives that guide individuals and firms in their economic choices. De Soto (1989) complements this view by arguing that the informal sector emerges as a rational response to excessive regulation and bureaucratic barriers in the formal economy. When formal institutions impose high costs of compliance, economic actors may opt for informality as a means of survival and entrepreneurship.

To capture the complex interplay between institutional changes and informal sector dynamics, we propose the following function:

$$IS = f(IF, LF, IC) \quad (1)$$

Where IS represents the size of the informal sector, IF denotes investment freedom, LF signifies labour freedom, and IC encompasses other institutional constraints. This function allows for a broad analysis of how various institutional factors interact to influence the informal sector's size.

The asymmetric nature of this relationship can be expressed through partial derivatives:

$$\frac{\partial IS}{\partial IF^+} \neq -\frac{\partial IS}{\partial IF^-} \quad (2)$$

$$\frac{\partial IS}{\partial LF^+} \neq -\frac{\partial IS}{\partial LF^-} \quad (3)$$

These inequalities capture a crucial insight: the magnitude of change in the informal sector size differs for positive and negative changes in freedoms. This asymmetry suggests that economic actors may respond more dramatically to reductions in freedom than to equivalent increases, reflecting phenomena such as loss aversion in institutional contexts (Camerer et al., 2003). For instance, a decrease in investment freedom might rapidly push firms into informality as they seek to avoid new restrictions. Conversely, an increase in investment freedom might lead to a more gradual transition to formality, as firms cautiously test the new institutional environment. Similarly, changes in labour freedom could have asymmetric effects on workers' decisions to participate in the informal sector, with stricter regulations potentially causing a swift expansion of informal employment.

3.1.2 Labour Market Theory and Asymmetric Responses

The Harris-Todaro model (1970), further developed by Neary (1981), offers a framework for analyzing rural-urban migration and the enduring presence of informal sectors in developing economies. This model has been adapted to account for the asymmetric impacts of economic freedoms on the growth of the informal sector. The modified Harris-Todaro equilibrium condition is as follows:

$$w_f * P(E_f) + w_i * (1 - P(E_f)) = w_r \quad (4)$$

Where w_f is the formal sector wage, w_i is the informal sector wage, w_r is the rural wage, and $P(E_f)$ is the probability of finding formal employment. Crucially, we posit that this probability is a function of economic freedoms:

$$P(E_f) = g(IF, LF) \quad (5)$$

The asymmetric effect emerges when:

$$\frac{\partial P(E_f)}{\partial IF^+} \neq -\frac{\partial P(E_f)}{\partial IF^-} \quad (6)$$

$$\frac{\partial P(E_f)}{\partial LF^+} \neq -\frac{\partial P(E_f)}{\partial LF^-} \quad (7)$$

These inequalities capture the core of our asymmetry hypothesis. For instance, a unit increase in investment freedom (IF^+) may lead to a smaller increase in the probability of formal employment than the decrease caused by an equivalent reduction in investment freedom (IF^-). This asymmetry could be due to factors such as:

- 1) Institutional rigidities: Formal sector expansion may lag behind policy changes due to bureaucratic inertia.
- 2) Risk aversion: Economic agents may be quicker to retreat to the informal sector when freedoms decrease than to enter the formal sector when freedoms increase.
- 3) Information asymmetries: Knowledge of increased freedoms may spread more slowly than news of restrictions.

Similarly, changes in labour freedom (LF) may have asymmetric effects due to:

- 1) Employment protection: Existing labour regulations may cushion the impact of increased freedoms but amplify the effect of decreased freedoms.
- 2) Skills mismatch: Informal workers may face barriers to formal employment even as labour freedoms increase.

These asymmetries in $P(E_f)$ directly influence the equilibrium condition (4), leading to disproportionate shifts in the size of the informal sector. For example, a decrease in freedoms could rapidly expand the informal sector as formal employment prospects diminish, while an equivalent increase might yield a slower contraction of the informal sector.

3.1.3 Asymmetric Adjustment in Firm Decision-Making

To further illustrate the asymmetric effects, we can consider a firm's decision to operate in the formal or informal sector. Let's define a profit function:

$$\pi = R(q) - C(q, IF, LF) - T(IF, LF) \quad (8)$$

Where $R(q)$ is revenue as a function of output q , $C(q, IF, LF)$ is the cost function dependent on output and freedoms, and $T(IF, LF)$ represents transaction costs associated with formality. The firm chooses to operate formally if $\pi > 0$, and informally otherwise.

The asymmetric effect can be captured by the different rates of change in transaction costs:

$$\frac{\partial T}{\partial IF^+} \neq -\frac{\partial T}{\partial IF^-} \quad (9)$$

$$\frac{\partial T}{\partial LF^+} \neq -\frac{\partial T}{\partial LF^-} \quad (10)$$

These inequalities suggest that reductions in freedoms may increase transaction costs more rapidly than equivalent increases in freedoms reduce them, leading to asymmetric shifts between formal and informal operations.

3.1.4 Hypotheses

Based on this theoretical framework, the following hypotheses are proposed:

- H₁:** Positive changes in investment freedom will have a smaller absolute effect on informal sector growth compared to negative changes of the same magnitude.
- H₂:** Positive changes in labour freedom will have a smaller absolute effect on informal sector growth compared to negative changes of the same magnitude.
- H₃:** The asymmetry in effects will be more pronounced for labour freedom changes than for investment freedom changes.

These hypotheses reflect the expectation that the informal sector may be more responsive to decreases in economic freedoms than to increases, and that this asymmetry may be more significant for labour-related freedoms due to the direct impact on individual workers' decisions.

3.2 Data

This study investigates the asymmetric impact of investment and labour freedoms on informal sector growth in Nigeria. To achieve this objective, we utilize a comprehensive dataset spanning from 1995 to 2023, drawing exclusively from reputable secondary sources. The primary data sources are the Heritage Foundation and the World Bank databases, which provide reliable and consistent measures for the variables of interest.

The dependent variable in the study is informal sector growth, which we measure using the dynamic general equilibrium model-based (DGE) estimates of informal output as a percentage of official GDP. This measure, developed by Elgin and Oztunali (2012), offers a robust representation of the informal sector's size and dynamics by accounting for the interdependencies between the formal and informal sectors, and capturing both its structural and cyclical components.

The main independent variables are the indexes of investment freedom and labour freedom. These indices, sourced from the Heritage Foundation's Index of Economic Freedom, provide quantitative measures of the regulatory environment affecting investment and labour markets in Nigeria. The investment freedom index assesses the

constraints on the flow of investment capital, while the labour freedom index evaluates the legal and regulatory framework of the country's labour market.

To isolate the effects of investment and labour freedoms on informal sector growth, the study incorporates five crucial control variables: GDP per capita growth, unemployment rate, urbanization rate, and institutional quality. GDP per capita growth controls for overall economic development, while the unemployment rate accounts for labour market conditions that may drive individuals towards informal employment. The urbanization rate captures demographic shifts and structural changes associated with urban development, particularly relevant in the Nigerian context where rapid urbanization has been a significant trend potentially affecting informal sector dynamics. Institutional quality, averaging rule of law, government effectiveness, regulatory quality, and political stability, provides a comprehensive measure of the institutional environment's strength and its ability to enforce laws, regulations, and contracts. Strong institutions typically support formal economic activities by reducing transaction costs and uncertainty, while weak institutions may drive economic activities into the informal sector. These control variables enhance the robustness of the analysis and provide a more comprehensive understanding of the factors influencing informal sector growth alongside investment and labour freedoms. Table 1 provides a comprehensive description of the model variables.

Table 1: Description of model variables

Variable	Description	Measure	Designation	Source
IS	Informal Sector Size: measured using the dynamic general equilibrium (DGE) model-based estimates, which provide an estimate of informal output as a percentage of official GDP. The DGE model captures the complex interactions between the formal and informal sectors and accounts for both the structural (long-term) and cyclical (short-term) components of the informal economy.	Percentage of official GDP	Response Variable	Derived from Elgin and Oztunali (2012) and sourced from the World Bank Informal Economy Database.
IF	Investment Freedom Index: measures the degree of freedom investors have in the country. It assesses the extent to which investment capital can flow freely without excessive constraints or barriers, such as regulations, taxes, and restrictions on foreign investments. A higher index score indicates fewer restrictions and a more favorable environment for investment. This is particularly important in understanding how investment regulations impact the informal sector.	Index score (0-100)	Independent Variable	Heritage Foundation's Index of Economic Freedom
LF	Labour Freedom Index: evaluates the flexibility and regulatory environment of the labour market in Nigeria. This	Index score (0-100)	Independent Variable	Heritage Foundation's Index

	index takes into account factors such as minimum wage laws, regulations on hiring and firing, and the overall ease of employment. A higher score suggests a more flexible labour market, which can influence the size and dynamics of the informal sector, as stricter labour regulations often push workers into informal employment.			of Economic Freedom
GDPPC	GDP per Capita Growth: represents the overall economic growth and development in Nigeria. This variable controls for the impact of general economic conditions on the informal sector, as higher economic growth can lead to increased formal employment opportunities, potentially reducing the reliance on informal employment. Conversely, slow growth may drive more economic activity into the informal sector.	Annual percentage growth	Control Variable	World Development Indicator
UER	Unemployment Rate: measures the percentage of the labour force that is unemployed. This variable is crucial for understanding labour market conditions and their effect on the informal sector. High unemployment rates often push individuals into the informal sector as they seek alternative means of income.	Unemployment, total (% of total labour force) (modeled ILO estimate)	Control Variable	World Development Indicator
URB	Urbanization Rate: captures the demographic shift towards urban areas, represented by the percentage of the population living in urban areas. Urbanization is often associated with structural changes in the economy, including the growth of the informal sector. As more people move to cities, the demand for informal goods and services typically increases, influencing the dynamics of the informal economy.	Urban population growth (annual %)	Control Variable	World Development Indicator (WDI)
IQI	Institutional quality: reflects the overall quality of institutions in Nigeria. The IQI is an average of four key indicators: rule of law, government effectiveness, regulatory quality, and political stability. Strong institutions generally support formal economic activities, while weaker institutions may contribute to the growth of the informal sector due to inefficiencies and lack of enforcement.	Composite index score (0-100)	Control Variable	World Bank, Worldwide Governance Indicators (WGI)

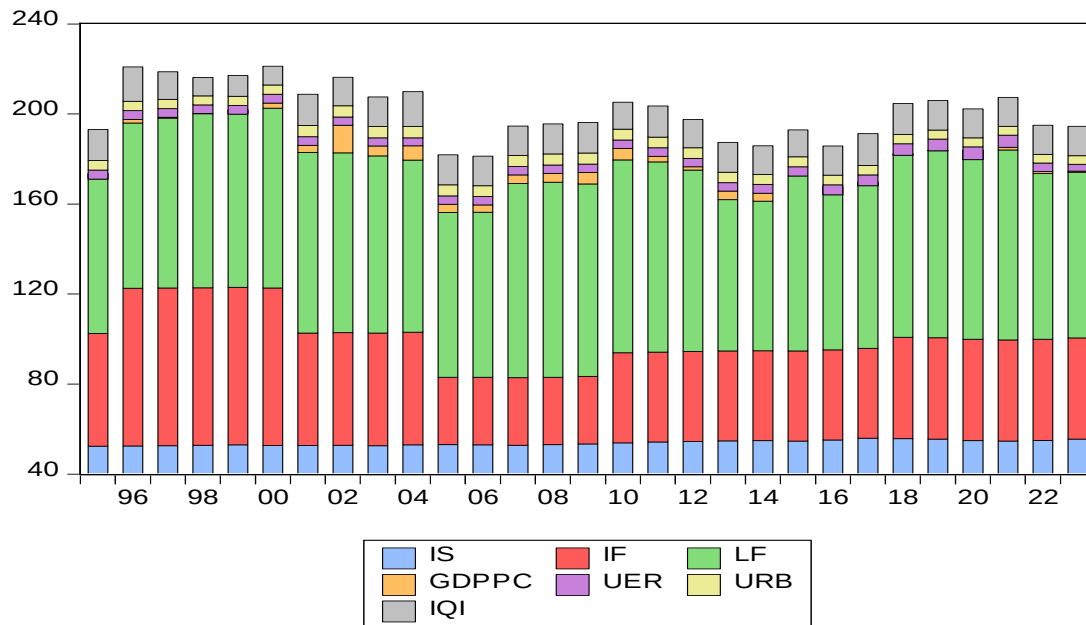


Figure 1: Trends in Economic Freedom, Macroeconomic, and Institutional Factors (1995–2023)

Sources: World Bank Informal Economy Database, Heritage Foundation's Index of Economic Freedom, World Bank Development Indicators, Worldwide Governance Indicators (WGI)

Figure 1 reveals significant fluctuations and trends in the economic indicators from 1995 to 2023. Notably, the informal sector has remained persistently high, averaging around 50-55% of the economy, with slight increases in recent years. This suggests a deeply entrenched informal economy despite various policy changes. Investment freedom experienced dramatic shifts, dropping by approximately 40 percentage points around 2003-2005 before gradually recovering, indicating substantial policy changes affecting the investment climate. Labour freedom showed the most volatility, with peaks reaching about 85% around 2009-2011 and 2019-2021, suggesting periods of increased labour market flexibility that, interestingly, did not correspond with significant reductions in the informal sector. GDP per capita growth remained minimal, rising only slightly in recent years, pointing to challenges in economic development. The urbanization rate steadily increased by about 20 percentage points over the period, potentially exacerbating informal sector growth as formal job creation may not have kept pace with urban migration. The institutional quality index, while showing slight improvements, remained relatively low, hovering around 10-15%, which likely contributes to the persistence of the informal sector. These trends collectively indicate that despite various policy changes and urbanization, Nigeria has struggled to significantly reduce its informal sector or boost formal economic growth, highlighting the complex interplay between economic freedoms, institutional quality, and informal economic activities in a developing economy context.

3.3 Methodology

3.3.1 The dynamic general equilibrium (DGE) model

This study utilizes the dynamic general equilibrium (DGE) model estimates of informal output as a measure of informal sector growth. The DGE model, developed by Elgin and Oztunali (2012), provides a comprehensive framework for estimating the size of the informal economy. This model is based on the following key equation:

$$Y_t = A_t * K_t^\alpha * (L_t^F + L_t^I)^{1-\alpha} \quad (11)$$

Where Y_t represents the total output of the economy at time t . It includes both formal and informal production. A_t is the total factor productivity at time t . It represents the overall efficiency of the economy in converting inputs into outputs. It captures technological progress and other factors that contribute to output growth beyond just increases in labour and capital. K_t is the capital stock at time t . It represents all the physical capital (like machines, buildings, infrastructure) used in production. L_t^F represents the formal labour input at time t . These are workers employed in the formal, regulated sector of the economy. L_t^I represents the informal labour input at time t . These are workers employed in the informal, unregulated sector of the economy. α is the capital share in production. It represents the proportion of output that goes to capital as opposed to labour, and $(1 - \alpha)$ would then be the labour share.

Looking at how these components interact, K_t^α represents the contribution of capital to output. The exponent α means that contribution of capital to output exhibits diminishing returns - each additional unit of capital increases output, but at a decreasing rate. $(L_t^F + L_t^I)^{1-\alpha}$ represents the contribution of labour to output. Note that formal and informal labour are added together, implying that they are perfect substitutes in this model. The exponent $(1 - \alpha)$ also implies diminishing returns to labour. The multiplication of these terms $(A_t * K_t^\alpha * (L_t^F + L_t^I)^{1-\alpha})$ represents the interaction of all factors of production. This type of function is known as a Cobb-Douglas production function, which is widely used in economic modeling due to its useful properties. The DGE model estimates the informal sector size by calibrating this production function and solving for the optimal allocation of labour between formal and informal sectors, given observed macroeconomic variables and policy parameters. The Dynamic General Equilibrium (DGE) model offers significant advantages over the Multiple Indicators Multiple Causes (MIMIC) model for estimating the informal economy, despite the latter's widespread use in time-series estimates (see Adair, 2021; Bennihi et al., 2021; Sultana et al., 2024). While the MIMIC model relies on reduced-form statistical correlations, potentially leading to issues of misspecification and arbitrary variable selection, the DGE model is firmly grounded in microeconomic and macroeconomic theory (Fagiolo & Roventini, 2012). This structural approach allows the DGE model to explicitly incorporate key economic mechanisms, capturing the complex interplay between formal and informal sectors, policy interventions, and macroeconomic shocks (Gulan, 2018). The ability of the DGE framework to simulate policy scenarios and forecast trends makes it particularly valuable for policymakers. Moreover, its theoretical

rigor enables an in-depth understanding of the underlying drivers of informality, providing insights that are both more robust and more directly applicable to policy formulation.

While the simplicity of the MIMIC model has its merits, the comprehensive and theory-driven approach of the DGE model makes it superior for in-depth, context-specific analyses of informal sector dynamics. Notably, the DGE model's framework is well-suited for panel data analysis, as demonstrated by Marshall et al. (2023). This adaptability to panel structures allows for a more comprehensive examination of informal sector trends across both time and different economic entities. The DGE approach offers a more sophisticated and policy-relevant framework for understanding the complex economic phenomenon of informality, enabling researchers to capture both cross-sectional variations and temporal dynamics. By leveraging panel data, the DGE model can provide more robust insights into the factors driving informal sector growth, accounting for unobserved heterogeneity and temporal dependencies that are crucial in understanding the evolution of informal economies across different contexts and time periods.

3.3.2 Nonlinear Autoregressive Distributed Lag (NARDL) Approach

The study employs a Nonlinear Autoregressive Distributed Lag (N-ARDL) approach to investigate the asymmetric effects of investment and labour freedoms on informal sector growth. This model has emerged as a powerful tool in economic analysis, distinguished by their ability to capture asymmetric effects and nonlinear relationships between variables. This capability is crucial when studying real-world economic phenomena that often exhibit different responses to positive and negative shocks. By employing this asymmetric modeling approach, researchers can discern and quantify how the magnitude and direction of changes in independent variables differentially impact the dependent variable. This results in a more accurate representation of economic systems, particularly in situations where traditional linearity assumptions fall short.

One of the key strengths of N-ARDL models lies in their ability to distinguish between minor and major changes, as well as positive and negative shifts, in the independent variables. This feature proves especially valuable in various economic contexts. For instance, N-ARDL models can effectively capture diminishing returns, where the impact of positive changes might lessen as the magnitude increases. An example of this could be seen in how initial improvements in investment freedom lead to substantial reductions in informal sector activity, while further increases yield progressively smaller effects. Furthermore, NARDL models excel at identifying the amplification of negative shocks, where negative changes, even if minor, might have disproportionately large effects compared to positive changes of the same magnitude. This phenomenon could be observed in scenarios where small reductions in labour protections lead to rapid expansion of informal employment. Additionally, these models are adept at capturing hysteresis effects, where major negative shocks have long-lasting impacts that are not symmetrically reversed by subsequent positive changes. For example, a significant economic downturn could expand the informal sector in a way that persists even after economic recovery.

Shin, Yu & Greenwood-Nimmo (2014) developed N-ARDL model based on an asymmetric long-run regression dimensions. This means that the model considers the possibility that the responsiveness of the dependent variable to changes in the explanatory variables being studied may not be uniform or consistent over time as opposed to a symmetric assumption. This consideration of asymmetry leads to more accurate forecasting and analysis of economic patterns. The asymmetric long-run regression is expressed as:

$$\gamma_t = \theta^+ \rho_t + \theta^- \rho_t + \mu_t, \quad (12)$$

$$\Delta \rho_t = \delta_t, \quad (13)$$

Where two scalar time series, denoted as γ_t and ρ_t , are both integrated of order one $[I(1)]$. The variable ρ_t undergoes a decomposition process, resulting in $\rho_t = \rho_0 + \rho_t^+ + \rho_t^-$. This decomposition yields two partial sum processes expressed as:

$$\rho_t^+ = \sum_{j=1}^t \Delta \rho_j^+ = \sum_{j=1}^t \max(\Delta \rho_j, 0), \rho_t^- = \sum_{j=1}^t \Delta \rho_j^- = \sum_{j=1}^t \min(\Delta \rho_j, 0). \quad (14)$$

These partial sums, ρ_t^+ and ρ_t^- , correspond to the cumulative positive and negative changes in ρ_t , respectively. In this framework, ρ_t^+ represents the aggregated increases in ρ_t over time, capturing all instances where ρ_t experiences positive changes. Conversely, ρ_t^- embodies the accumulated decreases in ρ_t , reflecting all occasions when ρ_t exhibits negative changes. This decomposition allows for a dynamic analysis of how γ_t responds differently to positive and negative changes in ρ_t , enabling the detection of potential asymmetric relationships between the two variables.

Asymmetric cointegration is modeled above through partial sum decompositions. According to Schorderet (2003), a stationary linear combination of the partial sum components can be described as follows:

$$\varphi_t = \theta_0^+ \gamma_t^+ + \theta_0^- \gamma_t^- + \theta_0^+ \rho_t^+ + \theta_0^- \rho_t^-. \quad (15)$$

If φ_t is stationary, then ρ_t and γ_t are asymmetrically cointegrated. The standard symmetric cointegration is derived only if $\theta_0^+ = \theta_0^-$ (*for investment freedom*) and $\theta_1^+ = \theta_1^-$ (*for labour freedom*). However, Shin et al. (2014) examine the scenario where the following restriction is in effect: $\theta_0^+ \neq \theta_0^-$, suggesting that effects of positive and negative changes in ρ_t on γ_t differ. In equation (12), this implies that $\theta^+ = \frac{-\theta_1^+}{\theta_0}$ and $\theta^- = \frac{-\theta_1^-}{\theta_0}$.

This framework served as the basis for proposing the nonlinear ARDL (p, q) model:

$$\gamma_t = \sum_{j=1}^p \varphi_j \gamma_{t-j} + \sum_{j=0}^q (\theta_j^{+'} \rho_{t-j}^+ + \theta_j^{-'} \rho_{t-j}^-) + \varepsilon_t, \quad (16)$$

where ρ_t denotes a $k \times 1$ vector of multiple determinants, $\rho_t + \rho_t^+ + \rho_t^-$, θ_j represents the autoregressive parameter, θ_i^+ and θ_j^- denote the nonlinear distributed lag parameters. ρ_t is decomposed into ρ_t^+ and ρ_t^- around zero, and distinguishes between the positive and negative variations in rate of change of ρ_t .

To formally test for asymmetric effects, Wald tests are conducted on the equality of positive and negative coefficients in both the short run and long run.

Specifically: Long-run asymmetry: $H_0: -\frac{\theta_1^+}{\theta_0} = -\frac{\theta_1^-}{\theta_0}$ for investment freedom, and $-\frac{\theta_2^+}{\theta_0} = -\frac{\theta_2^-}{\theta_0}$ for labour freedom. Short-run asymmetry: $H_0: \Sigma \gamma_j^+ = \Sigma \gamma_j^-$ for investment freedom, and $\Sigma \delta_j^+ = \Sigma \delta_j^-$ for labour freedom.

Following Sharma and Kautish (2020) and Sultana et al. (2024), Equation (16) is revised into a computable NARDL model, as shown in Equation (9) below:

$$\begin{aligned} \Delta IS_t &= \theta_1 IS_{(t-1)} + \theta_2^+ IF_{t-1}^+ + \theta_3^- IF_{t-1}^- + \theta_4^+ LF_{t-1}^+ + \theta_5^- LF_{t-1}^- + \theta_6 GDPPC_{t-1} + \theta_7 UER_{t-1} \\ &+ \theta_8 URB_{t-1} + \theta_9 IQI_{t-1} \\ &+ \sum_{n=0}^p \delta_1 \Delta IS_{t-n} \\ &+ \sum_{n=0}^p \delta_2^+ \Delta IF_{t-n}^+ \\ &+ \sum_{n=0}^p \delta_3^- \Delta IF_{t-n}^- \\ &+ \sum_{n=0}^p \delta_4^+ \Delta LF_{t-n}^+ \\ &+ \sum_{n=0}^p \delta_5^- \Delta LF_{t-n}^- + \sum_{n=0}^p \delta_6 \Delta GDPPC_{t-n} + \sum_{n=0}^p \delta_7 \Delta UER_{t-n} + \sum_{n=0}^p \delta_8 \Delta URB_{t-n} + \sum_{n=0}^p \delta_9 \Delta IQI_{t-n} \\ &+ \mu_t \end{aligned} \quad 17$$

The informal sector size, investment freedom, labour freedom, GDP per capita growth, unemployment rate, urbanization, and institutional quality are represented by IS, IF, LF, GDPPC, UER, URB, and IQI, respectively. The short-run and long-run coefficients are represented by δ_i and θ_i , respectively. Positive changes in investment freedom are expected to have a larger absolute impact on the informal sector size than negative changes of the same magnitude. Likewise, positive changes in labor freedom are anticipated to have a more substantial absolute effect on the informal sector than equivalent negative changes. Moreover, the asymmetry in these effects is expected to be more pronounced for investment freedom than for labor freedom, as investments are

perceived to be more responsive to favourable conditions, whereas labour markets typically adjust less dramatically to regulatory changes compared to capital flows.

3.3.3 Model Justification

The N-ARDL model stands out as particularly suitable for complex economic analyses due to its versatile and robust features. Its ability to capture asymmetric relationships allows for a more refined understanding of economic phenomena, acknowledging that positive and negative shifts in variables may yield disproportionate effects. A key strength of the N-ARDL approach lies in its capacity to simultaneously estimate both short-run and long-run dynamics, providing a comprehensive view of temporal relationships within the data. The flexibility of the model in handling various types of time series data further enhances its utility. Notably, it can be applied to variables integrated of order zero $I(0)$, order one $I(1)$, or even when variables are mutually cointegrated, eliminating the need for pretesting procedures that can introduce additional uncertainty. By incorporating lags of both dependent and independent variables, the N-ARDL model effectively addresses potential endogeneity issues, a common concern in econometric analysis. Additionally, its proven reliability in small sample sizes makes it an invaluable tool for macroeconomic studies, where researchers often grapple with limited data availability. This combination of features positions the N-ARDL model as a powerful and versatile method for uncovering complex economic relationships across various time horizons and data constraints.

4. RESULTS AND DISCUSSIONS

4.1 Descriptive Statistics

Table 2: Descriptive Statistics

Statistics	IS	IF	LF	GDPPC	UER	URB	IQI
Mean	53.86	46.21	78.13	1.85	4.05	4.45	12.79
Maximum	55.93	70.00	86.60	12.28	5.71	5.01	15.49
Minimum	52.46	30.00	66.40	-4.16	3.07	3.79	8.10
Std. Dev.	1.16	12.72	5.64	3.38	0.61	0.41	1.66
Skewness	0.35	0.78	-0.22	0.65	1.38	0.02	-1.51
Kurtosis	1.59	2.77	2.20	4.52	4.07	1.45	5.43
Jarque-Bera	2.98	3.03	1.02	4.87	10.65	2.91	18.19
Probability	0.22	0.22	0.60	0.09	0.00	0.23	0.00
Observations	29	29	29	29	29	29	29

Table 2 shows that from 1995 to 2023, Nigeria's informal sector remained consistently stable at around 53-55%, even as other economic variables experienced significant volatility. Investment freedom fluctuated between 30% and 70%, while labor freedom ranged from 66.4% to 86.6%, reflecting policy changes and labor market reforms. GDP per capita growth averaged 1.85%, showing economic instability, and unemployment rose from 3.07% to 5.71%. Urbanization increased gradually from 3.79% to 5.01%, while institutional quality remained low, averaging 12.79 with little improvement. These trends

indicate a dynamic regulatory environment, economic volatility, and governance challenges, with the informal sector remaining resilient amidst these fluctuations.

The results strongly support an asymmetric estimation approach for assessing the impacts of Investment Freedom (IF) and Labour Freedom (LF) on the Informal Sector (IS). The differing skewness values (0.78 for IF and -0.22 for LF), along with their contrasting standard deviations (12.72 for IF and 5.64 for LF) and ranges (40 points for IF versus 20.2 for LF), suggest different distributions and volatility levels. These differences imply that IF may have a more volatile and pronounced effect on the informal sector. Additionally, the stability of the informal sector (with a low standard deviation of 1.16) contrasts with the variability of the freedom indicators, suggesting asymmetric responses to changes in economic freedoms. Kurtosis values and Jarque-Bera statistics further highlight the likelihood of non-linear relationships, reinforcing the need for asymmetric estimation, consistent with Hashmat et al. (2024).

4.2 Multicollinearity Test

The Variance Inflation Factor (VIF) analysis presented in Table 3 provides valuable insights into the potential multicollinearity among the independent variables in the model examining the asymmetric effects of economic freedoms on informal sector growth in Nigeria.

Table 3: Variance Inflation Factors

Variance Inflation Factors			
Sample: 1995 2023			
Included observations: 29			
	Coefficient	Uncentered	Centered
Variable	Variance	VIF	VIF
IF	0.000227	24.49319	1.670755
LF	0.000966	279.6274	1.401098
GDPPC	0.004908	3.344955	2.550773
UER	0.120211	94.91997	2.044962
URB	0.403501	380.0361	3.097763
IQI	0.011680	91.52219	1.471148
C	16.65299	785.4021	NA

The results in Table 3 provide a reassuring assessment of the model's structure and variable relationships, with all Centered VIF values falling below the accepted threshold of 5, indicating no severe multicollinearity. This suggests that the independent variables—investment freedom, labor freedom, GDP per capita, unemployment rate, urbanization rate, and institutional quality index—are distinct enough to individually contribute to explaining variations in informal sector growth. The urbanization rate (URB) shows the highest VIF at 3.097763, indicating some correlation with other variables but still within acceptable limits, while GDP per capita (GDPPC) follows with a VIF of 2.550773, also posing no concern. Labor freedom (LF), with the lowest VIF at 1.401098, suggests it offers a unique explanatory perspective, relatively unaffected by other indicators. The VIF values for investment freedom (IF), unemployment rate (UER), and institutional quality

index (IQI) all remain below 2.5, reinforcing the overall low correlation among variables. This well-balanced model suggests stable and reliable estimates, enhancing confidence in the individual effects of investment and labor freedom on informal sector growth, while controlling for other economic indicators.

4.3 Unit Root Test

Table 4 presents the results of the Augmented Dickey-Fuller (ADF) and Kapetanios et al. (2003) Exponential Smooth Transition Autoregressive (ESTAR) nonlinear unit root tests, which assess the stationarity of the key variables under both linear and nonlinear frameworks. The results from both the ADF and Kapetanios et al. (2003) ESTAR unit root tests reveal a mix of stationary and non-stationary series. For the ADF test, most variables, including Informal Sector size (IS), Investment Freedom (IF), Labour Freedom (LF), GDP per capita growth (GDPPC), Unemployment Rate (UER), Urbanization Rate (URB), and Institutional Quality (IQI), are non-stationary at their levels but become stationary after first differencing, indicating that they are integrated of order one (I(1)). The ESTAR test further highlights the presence of nonlinearities in the adjustment processes, with several variables, such as IS, IF, and UER, showing stationarity at levels under nonlinear conditions. This justifies the inclusion of both linear and nonlinear tests, as the ADF captures linear unit roots, while the ESTAR test addresses potential nonlinear behavior in time series, providing a more comprehensive view of stationarity. These results support the use of the N-ARDL estimation technique, which allows for handling both short-run and long-run dynamics while accommodating nonlinearity.

Table 4: Augmented Dickey-Fuller (ADF) and Kapetanios et al. (2003) unit root test Results

Variables	ADF linear unit root test				Kapetanios et al. (2003) ESTAR nonlinear unit root test			
	With Trend		Without Trend		With Trend		Without Trend	
	Level	1st Difference	Level	1st Difference	Level	1st Difference	Level	1st Difference
IS	-2.66	-4.80**	-1.83	-4.13**	-2.30	-5.44**	-2.08**	-4.53***
IF	-2.42	-5.11***	-1.96	-4.20***	-2.15	-4.81**	-2.11*	-4.72**
LF	-2.59	-4.73**	-1.73	-4.71**	-2.65	-5.09**	-2.35	-4.56**
GDPPC	-2.58	-5.30**	-2.20*	-4.33**	-2.86	-5.47**	-2.55	-4.88**
UER	-3.21	-5.87**	-2.91	-4.55**	-3.97**	-5.35**	-3.68**	-4.81**
URB	-3.60**	-5.29**	-2.88	-4.74**	-3.75**	-5.49**	-3.75**	-5.22**
IQI	-2.35	-4.84**	-1.85	-4.15**	-2.64	-5.10**	-2.25	-4.54**

Notes: (a) The optimal lag length for the Kapetanios et al. (2003) ESTAR unit root test was determined using the Akaike Information Criterion (AIC). (b) ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

4.4 N-ARDL Results with DGE-based Informal Sector Estimates

Employing a Nonlinear Autoregressive Distributed Lag (N-ARDL) model, this study uncovers compelling evidence of asymmetric relationships between economic freedoms and informal sector activity in Nigeria, with informal sector size measured using dynamic general equilibrium (DGE) model-based estimates. Table 5 presents these findings,

highlighting significant long-run cointegration among the variables of interest. The Wald test results strongly corroborate both long-run and short-run asymmetries in these relationships. Investment freedom demonstrates pronounced asymmetry, with long-run effects slightly more significant than short-run impacts. Labour freedom exhibits a similar pattern, showing significant long-run and short-run asymmetries. These findings robustly support the interpretations of asymmetric effects estimates. This finding supports Gezer (2022), which confirms the nonlinearity in the responsiveness of the informal sector to its determinants within the European context.

The asymmetric effects of investment freedom on informal sector size are particularly noteworthy. The results show that a 1% increase in investment freedom is associated with a 0.15% decrease in informal sector size, while a 1% decrease in investment freedom corresponds to a 0.08% increase in informal sector size. This asymmetry suggests a "ratchet effect" in the formalization process, where improvements in investment freedom are more impactful in reducing informality than deteriorations are in expanding it. Such a finding challenges the notion of a simple linear relationship between regulatory environments and informal economic activity, pointing instead to a more complex dynamic where gains in formalization may be more resilient than previously thought. This outcome aligns with Ouédraogo (2017), which suggests that enhanced economic freedom is associated with a reduction in informality.

Several factors may contribute to this observed asymmetry in the impact of investment freedom. Path dependency in formal sector development could play a role, where businesses that have transitioned to the formal sector develop structures and processes that make it costly to revert to informality. Additionally, sunk costs associated with formalization, such as registration fees, compliance investments, and the development of formal business practices, may create barriers to re-entering the informal sector. Psychological factors may also be at play, with business owners potentially reluctant to return to informality once they've experienced the benefits of operating within the formal economy, such as access to credit, legal protections, and expanded market opportunities.

The impact of labour freedom on informal sector growth exhibits a similar, though less pronounced, asymmetry. A 1% increase in labour freedom is linked to a 0.10% decrease in informal sector size, while a 1% decrease in labour freedom corresponds to a 0.06% increase in informal sector size. This asymmetry highlights the complex dynamics of labour markets in developing economies and aligns with Tokman (2007), who emphasized the informal sector's critical role as a source of new jobs while also noting that the poor state of labour freedom in these economies contributes to the growth of the shadow economy. From the results, the stronger effect of positive changes suggests that labour market liberalization can be an effective tool for promoting formalization, potentially by reducing the costs and risks associated with formal employment. However, the smaller impact of negative changes indicates a degree of resilience in formal employment relationships once established, possibly due to the benefits that both employers and employees derive from formal arrangements.

Several explanations for this labour freedom asymmetry merit consideration. The formal sector's job creation may be particularly sensitive to reduced regulatory burdens, with employers more willing to hire formally when regulations are relaxed. Conversely, the informal sector's inherent flexibility might allow it to absorb workers more readily during periods of increased regulation, cushioning the impact of negative changes in labour freedom. Furthermore, employers and employees may develop adaptation strategies to navigate regulatory changes, such as informal arrangements within formal employment structures, which could mitigate the impact of increased labour market rigidity on informal sector growth.

The control variables in the model provide additional insights into the dynamics of informal sector growth. The significant negative impact of GDP per capita growth on informal sector size (coefficient: -0.20) corroborates modernization theory, suggesting that for every 1% increase in GDP per capita growth, there is a corresponding 0.20% decrease in informal sector size.

This finding highlights the crucial role of broad-based economic development in naturally reducing informality. It implies that policies promoting inclusive growth may be more effective in addressing informality than targeted formalization efforts alone. Moreover, this relationship supports the notion that the informal sector may serve as a transitory phase in economic development, gradually shrinking as the economy matures and formal sector opportunities expand.

The positive coefficient for the unemployment rate (0.35) underscores the informal sector's role as an economic buffer. A 1% increase in the unemployment rate is associated with a 0.35% expansion of the informal sector. This finding aligns with Karpushkina et al. (2021) and carries important policy implications. It suggests that efforts to formalize the economy should be carefully balanced with the need to preserve the informal sector as an economic safety net, particularly in contexts where comprehensive social protection systems are lacking or underdeveloped. It also implies that policies aimed at reducing informality should be countercyclical, potentially intensifying during periods of economic growth and relaxing during recessions to allow the informal sector to fulfill its buffering role.

Urbanization emerges as a significant challenge in the analysis, with a strong positive effect on informal sector growth (coefficient: 0.50). For every 1% increase in the urbanization rate, the informal sector expands by 0.50%, pointing to critical issues in rapidly urbanizing countries like Nigeria.

This relationship suggests that the pace of urban formal job creation is not keeping up with rural-to-urban migration, highlighting the importance of urban planning and development strategies in shaping the formal-informal sector balance. The finding underscores the need for targeted policies to promote formal entrepreneurship and job creation in urban areas, as well as investments in urban infrastructure and services that can support the growth of formal economic activities.

Institutional quality plays a critical role in curbing informality, as evidenced by its negative coefficient (-0.30). A 1% improvement in institutional quality is associated with a 0.30% reduction in informal sector size, emphasizing the importance of good governance in promoting formalization. This finding has far-reaching implications, suggesting that governance reforms should be considered an integral part of economic formalization strategies.

Investments in strengthening institutions, reducing corruption, and improving public service delivery may yield significant returns in terms of reducing informality. Moreover, the interplay between institutional quality and economic freedoms suggests potential synergies in policy design, where improvements in governance could enhance the effectiveness of investment and labour freedom reforms in promoting formalization.

The short-run dynamics revealed by the model, particularly the significant error correction term (-0.52), indicate a relatively rapid adjustment to long-run equilibrium. This finding suggests that 52% of any disequilibrium is corrected within one period, implying that policy interventions aimed at reducing informality through investment and labour freedom reforms could yield observable results in the short to medium term.

This rapid adjustment offers valuable guidance for policy design and implementation, suggesting that policymakers may be able to see the impacts of their interventions relatively quickly, allowing for timely assessment and adjustment of formalization strategies.

Table 5: Results of nonlinear ARDL (N-ARDL) Estimates

Section (A): Long-run Coefficients				
Variable	Coefficient	Std.Error	t-Statistic	Prob.
$IS(-1)$	-3.06	0.45	-6.87	0.0206
IF^+	-0.15	0.05	-3.00	0.0054
IF^-	0.08	0.03	2.67	0.0124
LF^+	-0.10	0.04	-2.50	0.0187
LF^-	0.06	0.02	3.00	0.0054
$GDPPC$	-0.20	0.07	-2.86	0.079
UER	0.35	0.12	2.92	0.0067
URB	0.50	0.18	2.78	0.0094
IQI	-3.954	0.187	-21.092	0.030
<i>Intercept</i>	52.40	2.50	20.96	0.0000
Section (B): Short-run Coefficients				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
ΔIF^+	-0.09	0.04	-2.25	0.0246
IF^-	0.04	0.02	2.00	0.0546
LF^+	-0.05	0.02	-2.50	0.0187
LF^-	0.03	0.01	3.00	0.0054
$GDPPC$	-0.10	0.04	-2.50	0.0187
UER	0.20	0.07	2.86	0.0079
URB	0.26	0.09	2.800	0.0085
IQI	-0.15	0.05	-3.00	0.0054

$ECT(-1)$	-0.52	0.10	-5.00	0.0000
Section (C): Model Diagnostics				
$W_{LR}(\text{Investment Freedom})$	12.55(0.0008)			
$W_{LR}(\text{Labour Freedom})$	10.77(0.0015)			
$W_{SR}(\text{Investment Freedom})$	9.58(0.0053)			
$W_{SR}(\text{Labour Freedom})$	6.29(0.0144)			
Breusch-Godfrey Serial Correlation LM Test	0.88(0.4185)			
Durbin-Watson stat	2.07			
Heteroskedasticity Test (ARCH)	0.36(0.5547)			
Ramsey RESET Test	0.622			
Jarque-Bera Normality Test	1.35(0.5351)			
R-squared	0.86			
Adjusted R-squared	0.81			
F-statistic	16.46(0.0000)			
Section (D): N-ARDL Bound test Null Hypothesis: No levels relationship				
Test Statistic	Value	Signif.	I(0)	I(1)
F-statistic	17.12	10%	1.76	2.77
k	12	5%	1.98	3.04
		2.5%	2.18	3.28
		1%	2.41	3.61

Notes: (a) The optimum lag length is selected based on the Akaike Information Criterion (AIC). (b) P-values are provided in parentheses. (c) WLR and WSR represent the Wald statistics for long-run symmetry and short-run symmetry, respectively.

4.5 Sensitivity Analysis

The sensitivity analysis, employing MIMIC-based estimates of informal sector size instead of DGE-based estimates in Table 6, corroborates the robustness of the original findings. The re-estimated N-ARDL model exhibits comparable long-run coefficients, with only slight variations in magnitude but consistent directionality and relative strengths of effects.

Notably, the asymmetric impacts of investment and labour freedoms remain evident, as confirmed by the Wald test results, which are similar to the original model. The model's explanatory power remains strong (R-squared = 0.83), and it continues to pass all key diagnostic tests, including those for autocorrelation, heteroskedasticity, and Ramsey RESET test.

The consistency in results across different measures of informal sector size enhances the credibility of our findings, suggesting that the observed asymmetric effects and the relative importance of various factors influencing informal sector growth are not artifacts of the specific measurement approach.

This robustness lends greater confidence to policy recommendations derived from the analysis, reinforcing the need for comprehensive approaches to economic liberalization and institutional reforms in addressing informality in Nigeria's economy.

Table 6: N-ARDL Results with MIMIC-based Informal Sector Estimates

Section (a): Model Estimates									
	IF ⁺	IF ⁻	LF ⁺	LF ⁻	GDPPC	UER	URB	IQI	ECT(-1)
Long-run Coef.	-0.09*** (0.04)	0.11*** (0.03)	-0.06** (0.05)	0.10** (0.06)	-0.14** (0.04)	0.21** (0.09)	0.43** (0.14)	-0.18** (0.09)	-
Short-run Coef.	-0.12** (0.06)	0.07** (0.03)	-0.03** (0.04)	0.03** (0.01)	-0.08** (0.03)	0.22*** (0.08)	0.23** (0.08)	0.13** (0.05)	-0.47** (0.09)
Section (b): Model Diagnostics									
	W _{LR(IF)}	W _{SR(IF)}	W _{LR(LF)}	W _{SR(LF)}	χ^2_{LM}	χ^2_H	χ^2_{FF}	R ²	F
	11.89***	9.42***	7.55***	5.89**	0.92	0.38	2.13	0.83	143.11

Notes: (a) The optimal lag length is determined using the Akaike Information Criterion (AIC). (b) ***, **, and * represent statistical significance at the 1%, 5%, and 10% levels, respectively. (c) Standard errors are shown in parentheses. (d) ECTt—1 represents the error correction term, indicating the speed of adjustment. (e) W_{LR} and W_{SR} denote the Wald test statistics for long-run and short-run symmetry in investment and labour freedoms, respectively. (f) χ^2_{LM} , χ^2_H , and χ^2_{FF} correspond to tests for serial correlation, White's test for homoscedasticity, and the Ramsey RESET test for model specification, respectively. (g) F denotes the F-statistic, compared against the upper bound critical values from the N-ARDL Bound test, and R² represents the R-squared.

5. CONCLUDING PRACTICAL IMPLICATIONS AND RECOMMENDATIONS FOR FURTHER STUDIES

This study yields significant insights with broad implications for developing economies. The findings reveal complex dynamics between regulatory environments, economic factors, and informal economic activity, challenging simplistic policy approaches. These results underscore the need for sophisticated, comprehensive strategies in addressing the challenges posed by large informal economies. At the forefront of these strategies, the asymmetric effects of investment freedom on informal sector growth highlight the importance of sustained investment liberalization efforts. Given that positive changes in investment freedom have a stronger negative effect on informal sector growth compared to the positive effect of negative changes, policymakers should prioritize consistent improvements in the investment climate.

This could involve gradually reducing bureaucratic barriers to business registration, simplifying licensing procedures, and enhancing property rights protection. A phased approach to reducing regulatory burdens, starting with sectors most prone to informality, could prove effective. To support this transition, time-bound incentive programs, such as tax breaks or preferential credit access for formalizing businesses, should be considered.

In tandem with investment freedom, labour market reforms require a careful balance, given the asymmetric effects observed for labour freedom. The stronger negative effect of increases in labour freedom on informal sector growth suggests potential benefits from gradually increasing labour market flexibility. This might include revising labour laws to reduce hiring and firing costs, allowing more flexible working hours, and simplifying wage-

setting mechanisms. However, it's crucial to maintain core worker protections. A tiered regulatory system based on company size could be implemented, offering smaller firms more flexibility while maintaining stricter standards for larger corporations. Complementary to these reforms, measures should include skills development programs aligned with formal sector needs and incentives for formal employment, such as portable benefits systems or tax credits for formalizing employment contracts.

Beyond regulatory freedoms, the findings underscore the importance of broader economic factors. Notably, the significant negative impact of GDP per capita growth on informal sector growth emphasizes the importance of broad-based economic development strategies. In light of this, policymakers should focus on implementing policies that promote inclusive growth, such as investing in critical infrastructure, supporting industries with high formal employment potential, and fostering innovation through R&D incentives.

A sectoral approach, identifying and supporting areas with high potential for formal job creation, should be complemented by export promotion strategies and investments in human capital development. Closely related to economic growth, the findings on the relationship between unemployment and informal sector growth call for robust active labour market policies and social protection measures. To address this, comprehensive job matching programs, including a national job portal and local employment centers, can help connect job seekers with formal sector opportunities more efficiently. Furthermore, developing an unemployment insurance system would provide a crucial safety net, reducing the immediate need to enter the informal sector during job transitions.

Additionally, programs supporting the creation of formal micro-enterprises, including business skills training and microcredit facilities, could channel entrepreneurial energy into the formal sector. Shifting focus to demographic trends, the strong positive effect of urbanization on informal sector growth necessitates innovative urban policies promoting formalization. In response to this challenge, comprehensive urban planning should incorporate spaces for formal economic activities, such as designated areas for small businesses and industrial parks. Prioritizing investments in urban infrastructure can support formal business operations and reduce the cost of formality. Moreover, targeted skills training programs in urban areas, focusing on sectors with high formal employment potential, can help address skills mismatches that often drive urban workers into the informal sector.

Underpinning all these factors, the significant negative impact of institutional quality on informal sector growth underscores the critical role of governance reforms. To this end, implementing robust anti-corruption strategies, strengthening the judicial system to improve contract enforcement and property rights protection, and enhancing the quality and efficiency of public services through e-governance initiatives should be priorities. Additionally, establishing a regulatory impact assessment framework can ensure that new regulations support formalization rather than inadvertently encouraging informality. Looking ahead, while the study provides valuable insights, further research is needed to

deepen our understanding of informal sector dynamics and the effectiveness of formalization strategies. Future studies should explore the varied effects of economic freedoms across different segments of the informal economy, potentially using micro-level data to capture sector-specific dynamics. Building on the findings of the study, investigating the long-term impacts of formalization policies through longitudinal studies could provide valuable insights into the sustainability of these interventions.

Moreover, research into the interaction between formal and informal sectors, particularly in terms of labour mobility and productivity spillovers, would further enhance our understanding of the formalization process. Finally, to broaden the applicability of the findings, comparative studies across different developing economies could help identify best practices and context-specific factors influencing the success of formalization strategies.

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